

**STUDY GUIDE
FOR
MATH ON THE MALL
— AND BEYOND**

**ESCHER PRINTS, RARE BOOKS, AND MUCH
MORE**

**FIELD TRIP
FOR
GEOMETRY AND ALGEBRA STUDENTS**

NOTES for the Rare Book Tour:

Only small groups can be admitted to the study room on the third floor of the East Wing of the National Gallery to view the special exhibition.

We will see works by graphic artist M.C. Escher that curator Greg Jecmen was kind enough to put together for us, and view the special collection of rare first editions of very old books on math and science from the Dibner Library that has been put on display especially for us by Ronald Brashear, the Head of Special Collections and Curator of Science & Technology Rare Books of the Smithsonian Institution Libraries.

So nobody will need to be waiting outside in a hallway for even 10 minutes while another group is inside, everybody is going to be split up into groups of about 10 people, and each group will follow slightly different paths.

Each group will see more or less the same things, but not necessarily in the same order. Thus, you should not necessarily do all of the items on this study guide in the order that they are printed here. Instead, please refer to your group's itinerary.

A. Fort Reno Park and Tenleytown

The path you are walking on from Deal crosses Fort Reno Park, the highest point of land in Washington, DC at 492 feet above sea level. Today it is mostly used as a reservoir in the water system (Why would that be?), as a playing field for various sports, and as a place to put antennas. (There is an unconfirmed rumor that it's a radio 'listening post' for some secret government agencies ...) But it has an interesting past.

Questions:

The Washington Monument is 555 feet tall, and stands on ground that is about 20 feet above sea level. So which is higher – the Fort Reno summit, or the top of the Washington Monument? And by how much?

If you add on the height of the towers and antennas at the summit of Fort Reno Park, then who's ahead?

Why would it be beneficial to have a reservoir and water tower in a part of town that is well above sea level?

After the disastrous battle of First Bull Run/First Manassas in the first year of the Civil War, the Union forces realized that the war was not going to be won in a hurry, and recognized the need to defend the nation's capital from the Confederates. Thus, they built fortifications all around the city, even in Virginia, that were placed on high ground for good strategic and tactical military reasons. The hill you are on was fortified by a regiment from Pennsylvania, and eventually the fortifications held a number of large cannons and about 3,000 men. It was renamed for General Jesse Reno, who was killed at the decisive Union victory at South Mountain, Maryland – a battle that directly led to the Union victory at Antietam a few days later (September, 1862). The fact that the Confederates were defeated at Antietam gave President Lincoln the impetus to sign the Emancipation Proclamation, which literally changed the character of the war. It also prevented European powers like France and England from recognizing the Confederacy as a sovereign nation, which would have been a disaster for the cause of the Union.

Two years later, soldiers at Fort Reno were the first to notice that Confederate forces led by Jubal Early were advancing on Washington while General Grant was using most of the rest of

his army to attack Petersburg, Virginia – the only time that the nation’s capital came under attack. Most of the fighting took place at Fort Stevens, which is a few miles to the east, near what we now call Georgia Avenue, but some shells were fired from Fort Reno at the Confederate attackers, who were defeated. President Lincoln actually came under fire during the battle, as he stood on top of the trenches at Fort Stevens watching the battle!

After the Civil War ended, all of the forts were abandoned, and freedmen (slaves who had won their liberty), black Union ex-soldiers, and their families occupied the area you are walking through, and built a community that was largely African-American, known as Tenleytown. The lonely concrete steps, manholes, and fire hydrants that you see in various spots in the park are from that time. The Rose School – the little gray stucco building that is in back of Deal JHS – was in the “Colored” section of the DC Public Schools at that time, and served that community. You can see the streets of Tenleytown on older maps of DC.

What happened to this community? Well, in the 1930’s a movement arose known as the “City Beautiful”, whose aim was, naturally, to beautify the city. But to them, that meant removing the lower classes – and in particular, African-Americans – from desirable properties near where middle-class whites wanted to live. This area of upper Northwest was one of those areas. This movement had a lot of support from the powers-that-be, as you can imagine, and succeeded in many, but not all, of their aims. They had three whites-only schools built – Murch, Deal, and Wilson; they had the black community on the Fort Reno summit removed (probably using the law of eminent domain) and the area turned into a park; and they had Nebraska Avenue and other streets built. They intended to connect all of the former forts that surrounded Washington with a road that was to be called Fort Drive. Here they did not succeed – the only “Fort Drive” that I know of is the two-block-long lane right here in front of Deal.

Mr. Brandenburg attended Deal from 1961 to 1964, and the school looked very much like it does now, except there weren’t any fences around the playing fields, and that he does not recall any African-American students attending Deal during that time, though perhaps there might have been one or two. Segregation had been ruled illegal by the U.S. Supreme Court in the mid-1950’s, but it took the work of local activists like Julius Hobson to make desegregation a reality here in DC, in the late 1960’s and early 1970’s.

And those towers just uphill from the school? They look like they are out of the Middle Ages, but they are not, and they have nothing to do with the Civil War, either. All they are is camouflage for some water towers – the big containers of many thousands of gallons of water that keep the pressure up at your local tap or fire hydrant even if the pumps fail. Occasionally things go wrong with the pumping sensors, and the water towers overflow, flooding Fort Drive. Metal water towers are really ugly, as you know if you have ever seen one; the stonework dates from the 1930’s and is just there to make the City Beautiful. The towers even won some architectural awards.

B. ESTIMATION AND PARKING METERS

The street you are crossing – Chesapeake Street, NW, between Wilson SHS and Fort Reno Park – looked a little different five years ago. The biggest difference? The parking meters. Then they were a different model, and, five years ago, just about every single meter on this street had been smashed by criminals, and all of the money inside of each one had been stolen. That scene – every single parking meter broken and burglarized – was repeated on nearly every block across the city. (That symbol of urban crime and mismanagement was one of many pieces of bad news that propelled voters to elect an entirely different sort of mayor to try to clean things up.) Sources differed as to whether those smashing the meters were individuals or semi-organized crime. In any case, someone had discovered that a sledgehammer or similar device would work to break open the glass at the top of the parking meter and allow the perpetrator to steal all the coins inside.

The meters that are in place currently are digital, and so far have been impervious to the same sort of destruction and theft.

How much did all of that theft cost the city in lost parking revenue? Let's do some estimation to find out.

1. How many minutes of parking time do you get for a quarter? _____
 That means how much money for an hour? _____
 During which hours of the day do you have to pay for parking? _____
 During what percent of those hours do you estimate that there is actually somebody parked legally at one of these meters (i.e., is feeding the meter)? _____
 Use this information to estimate the income per day for the city for one parking meter on this street on one day: _____

(answer #1)

2. How many days of the week is it necessary to pay for parking? _____
 There are _____ weeks in a year, so that means that you have to pay for parking at these meters for about _____ days per year.
 However, there are roughly _____ federal holidays during which parking is free, so subtract that from your last answer to get a total number of days of paid parking per year:

(answer #2)

3. To find the total revenue from one parking meter on this street per year, simply multiply answer #1 by answer #2. Feel free to round either number off to make your computation easier. Result? _____ income per meter per year.

(answer #3)

4. Estimate how many parking spaces there are on one block (both sides of the street):

(answer #4)

5. Obviously, not all streets in the District of Columbia have parking meters. Generally, blocks near commercial districts (like Wisconsin Avenue, K Street NW, H Street NE, Connecticut Avenue and so on) have them, as do neighborhoods very close to Metro rail stations. Make a rough guesstimate of how many blocks you think have parking meters in the District of Columbia: _____ And about how many meters do you think there are on the average block?

(You can use answer #4, or else adjust it upwards or downwards, as you see fit.)

If you multiply those last two numbers, rounding to make the math easier, you should get an estimate for the total number of parking meters in the District of Columbia:

_____ (answer #5)

6. A Washington Post editorial of May 28, 1997 states that there were 15,000 parking meters in DC at that time, and of those, 3,000 had been destroyed or removed in the previous year. How close was your estimate for the total number of meters in DC to that of the Washington Post?

7. Use your estimate of the total revenue per parking meter (answer #3) and the Post's estimate of 3,000 destroyed meters to give a dollar amount for the total amount of lost revenue per year in vandalized meters: _____

8. Any other thoughts you might want to add:

C. METRO RAIL: GEOMETRY AND ESTIMATION

You will take Metro Rail from Tenley Circle/American University station to the Archives/Navy Memorial station, transferring to the Green or Yellow line at Gallery Place/Chinatown. While you are waiting on the platform, or riding the train, let's do a little math.

1. The platforms are covered with hexagonal, red-brown tiles that are all alike. They form one of the three regular (or 'pure') tessellations that exist. What is its numerical code?

2. However, near the edge of the platform, this tessellation of brown tiles gives way to another tessellation made with rubber squares that have little raised bumps on them. These squares form one of the other two regular tessellations. What is its numerical code?

3. Does anybody in your group know why Metro installed the rubber square tiles a few years ago, after initially complaining that they would cost too much? _____ If so, what is the reason?

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4. Metro's rail system is one of the deepest in the world, and the Tenley station is one of the deepest we have in DC. Much (but not all) of the Washington Metrorail system was actually built by mining – in other words, huge machines bored more-or-less cylindrical tunnels through bedrock or through gravel, clay, and other unconsolidated sediments, far underground. This is an expensive method, of course, but it prevented massive disruption of many streets. Other well-known subway systems, like the ones in London, Paris, New York, Chicago and Philadelphia were mostly built by 'cut-and-cover' methods. In that method, the builders would simply cut open existing streets, dig down and remove all the dirt, lay the foundations, build the tracks and stations, then build a roof for the tunnels and stations – usually supported by numerous metal columns – then cover it all over again with dirt. Then, the gas lines, water lines, sewer lines, and electric lines would be placed above the subway, and finally the pavement would be rebuilt. During the construction, of course, life would be a nightmare for all of those who lived along the path of the line. {Some of you may remember similar things happening along the recently-opened Green Line in the U Street area between downtown and Florida Avenue. If you want to read about details of this sort of construction, you might want to read *Underground*, by David Macaulay. And if you compare DC's subway to those in the other cities I mentioned, by actually visiting them and trying them out, I think that you will find that ours is much, much nicer in just about every possible respect. Does anybody in your group care to disagree?

5. Almost all of the underground portion of Metro is at or below the water table. Some of you may live in houses with leaky basements – Metro’s problems are similar, but larger. The section of the Red Line from Farragut North to the Bethesda Medical Center was not built with a plastic lining, because the engineers thought that the bedrock would not carry enough water to make leaks in the concrete in the tunnels. Unfortunately, they were wrong. At some stations, and between stations, you can see leaks cause things that look a lot like stalactites and stalagmites! Metro has to spend millions of dollars on a regular basis replacing rails, insulators, and wires, and trying to fix leaks. (See Washington Post, July 13, 2000)
6. But is it worth it? Let’s do some estimation of how many cars are taken off the streets each day just on the Red Line.

First of all – about how many rail cars are there on the average train? _____

And how many seats are there on one of these cars? _____

Almost all of the seats each hold two people, so that’s _____ seated passengers per car.

Then how many more people generally fit in, standing, during rush hour? _____

So that’s a total of about _____ passengers per car. Multiply that by the number of cars in a train, and you get _____ people moved per Metro train.

Now – about how much time elapses between the arrival of one train and the arrival of the next, on the average, during rush hour? (Don’t just count days when everything gets all screwed up, or the days when everything works better than you believe possible – take an average.) _____

So that’s about _____ trains per hour, in one direction. Rush hour is about _____ hours long.

Thus, during rush hour, the Red Line carries roughly _____ people.

7. If it weren’t for Metro, some of those people would take the bus, or a taxi. But a large proportion of them would attempt to get to their destination in private automobiles. What percent of them, in your opinion, would go by car if Metro hadn’t been built, or were to suffer a catastrophic breakdown? _____

So that’s roughly _____ people trying to get places during rush hour by car.

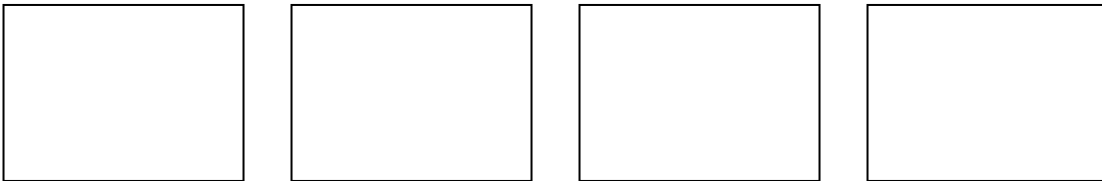
8. Now, about how many people do you think there would be per automobile?

_____ That means that having Metro takes about _____ cars off the DC streets, just in the downtown direction of Metro, every rush hour. Is it worth it, in your opinion?

D. U.S. NAVY MEMORIAL: CODES AND SPHERICAL GEOMETRY



















When you leave the Navy Memorial/Archives Metro rail station, you are right next to the Navy Memorial, which has a number of mathematical/geographical points of interest. First of all, the flags that are on the flagpoles, that are built to resemble the masts of modern naval ships, actually have a meaning that is utterly unknown to the ordinary person, but have very specific international naval meanings. In other words, they are in code. Use the code list on the next page to decode them.

1. Sketch the 4 naval signal flags that have no writing on them and whose meaning is not obvious. If you don't have colored pencils, you can use letters to stand for the colors – for example, B for blue, K for black, W for white, Y for yellow, G for green, R for red, and so on. Use the rectangles provided:



2. Now try to decode the message. Recall that each flag can have two meanings. Thus, the flag for the letter "F" – a red diamond (rhombus) inside a white rectangular flag, with the 4 vertices of the rhombus at the midpoints of each side of the flag – can either just mean the letter "F" or it can mean an entire message – "I am disabled, please communicate with me!" What does the entire message appear to mean?

Here is a summary of the meanings of those flags:

A		I have a diver down, Keep well clear.	O		Man overboard.
B		I have dangerous cargo.	P		All personnel return to ship.
C		Yes; affirmative.	Q		All boats return to ship.
D		I am maneuvering with difficulty.	R		Preparing to replenish.
E		I am directing my course to starboard.	S		Conducting flag hoist drill.
F		I am disabled. Communicate with me.	T		Do not pass ahead of me.
G		I require a pilot.	U		You are running into danger.
H		I have a pilot on board.	V		I require assistance.
I		Coming alongside.	W		I require medical assistance.
J		I am on fire and have dangerous cargo.	X		Stop carrying out your plans and watch for my signals.
K		I wish to communicate with you.	Y		I am dragging anchor.
L		Stop your vessel immediately.	Z		I need a tugboat.
M		My vessel is stopped, making no way.			
N		No; negative.			

3. Your turn – write your own first name using the same alphabetical code, with a maximum of 5 letters. By the way, a ship only carries a single set of flags, and some of them can be used to indicate a repetition of a previously-used letter. So if your name were Gigi, you would have to raise the flag for a G, then an I, then another flag that says to repeat the first letter, and finally another flag that says to repeat the second letter. Your name (or your partner's):

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4. Now use the other meaning of those naval signal flags, to send an entire message of your choice with just a single flag:

Your intended meaning:

5. Now look at the large map of most of the planet that is on the ground, made with two different types of stone. Notice that there are no labels, so you have to figure out where you are by the shape of the land masses or bodies of water.

What color stone represents water? _____
 What color represents land? _____
 And what do the curved lines stand for?

6. Let us find various places on that map. Check off the items that you actually do find. North America: _____ Long Island: _____ Chesapeake Bay: _____ Washington, DC (approximately): _____ Florida: _____ Spain and Portugal: _____ The Mediterranean Sea: _____ Morocco: _____ Italy: _____ The North Pole: _____ Greece: _____ Japan: _____ Alaska: _____ Brazil: _____ The Panama Canal: _____ The Suez Canal: _____

7. Which continent(s), if any, are not marked on the map at all?

8. You probably know that the earth is just about spherical (shaped like a ball). You may have learned that Christopher Columbus was the first person to figure this out, and that everybody else before his time thought that it was flat. Actually, that is a fable, or myth (or a lie, if you want to be blunt about it) that was invented by Washington Irving around 1830. Irving was the author of many other famous works of fiction, such as “The Legend of Sleepy Hollow”. In fact, nearly 1800 years ago, Ptolemy wrote a book on astronomy and geography, known as *The Almagest* (corrupted Latin and Arabic, meaning “The Greatest”) that plainly stated that the world was spherical. Ptolemy’s book was studied by all astronomers and navigators for well over 1300 years. {You will see a copy of a translation of Ptolemy’s *Almagest* printed in 1511 just outside the Dibner Library at the Smithsonian’s Museum of American History (S-MAH) during this tour.} Also, the well-known Greek librarian, mathematician and geographer Eratosthenes assumed that the world was spherical, and used simple geometry to calculate the size of the earth about 200 BC. His answer may have been very close to what we think it is today – about 25,000 miles around. {Hopefully, we will be able to repeat his experiment later this year.} What Columbus believed, and what he was totally wrong about, was that it was possible to sail westward from Europe and reach China or Japan in less than 2 months on the tiny little sailing boats of his day. Columbus thought that the earth was much smaller than what either Ptolemy or Eratosthenes thought, and he also thought that Asia stretched out eastward much farther than it actually does. Portuguese sailors and navigators had sailed nearly all around the entire continent of Africa by this point, and they thought Columbus’ plan was crazy – and they were right. It would have taken a very, very long time to sail westward to reach Asia, had the continents of North and South America not been in the way. Those ships averaged maybe 5 to 10 miles per hour, so the voyages would have been so long that everybody on board would have died of hunger, thirst, or scurvy. When Columbus reached land in October, 1492, he was absolutely convinced that he had reached some islands just off of the continent of Asia. Therefore, he called the natives “Indians” because he figured that they must be from India. By the time he died, he was still completely unaware that he had just made one of the biggest geographical discoveries of all time (at least as far as the major civilizations of the ‘Old World’ were concerned) – that there was in fact an entire new continent way out there at the end of the Atlantic Ocean. It took Amerigo Vespucci to put two and two together, and to proclaim that we here live in a New World. Consequently, these continents were named in the honor of Vespucci, and not Columbus. {And whether Columbus was a ‘nice guy’ to the ‘Indians’ that he met – you probably have an opinion on that already.}

9. You may or may not have learned in a geography class that it is impossible accurately to represent the surface of a globe on a flat surface such as a floor or a piece of paper. That is indeed true. In fact, it can be demonstrated, using advanced projective geometry that you might take in college if you are a mathematics major, that if you try to make a flat map of a globe, you can only preserve one out of the three following qualities: area, angle, and length. Thus, all flat maps of the earth are distortions, and require a lot of compromises. The map on the ground in the center of this plaza is said to be the largest grid map in the world. In any case, in my opinion it distorts things *less* than most maps, and it allows you to estimate the best (shortest, most direct) air paths between locations in North America and locations elsewhere. Let us try a few of these – you may find the results to be quite surprising.
10. Have one person stand on Washington, DC, and another person stand on London, England (the southeast corner, more or less, of that island nation). A direct path from one to the other does NOT go directly east from DC, out over the Atlantic. Instead, it goes over what parts of the USA?
 _____ and what parts of Canada? _____.
11. Now have somebody stand on Tokyo (one of the bays on the southern edge of the largest of the four main islands of Japan), while somebody stands on DC. Again, the direct path from one to the other is surprising – it goes nowhere near California and the bulk of the Pacific Ocean! In fact, it goes over which famous North American bodies of water?
 _____ and which much-less famous Canadian bodies of water (if anybody remembers their names) _____ and over which very large American state _____ and over what very-surprising parts of Asia _____ located in what nation?

12. Another surprising thing – the Panama canal. You probably know that it connects the Caribbean Sea with the Pacific Ocean, and it does. You probably think that it goes more or less east to west, with the eastward end on the Caribbean and the westward end on the Pacific. Look carefully – that is not true at all! What direction does it go in – in other words, if you want to travel by boat on the Panama Canal from the Caribbean to the Pacific, in which direction do you actually travel?

13. Yet another surprising geographical fact. Most of us think that since the Pacific Ocean is thousands of miles to our west, and South America is thousands of miles to our south, that if you want to fly in a straight line to, say, Chile or Peru – both located on the Pacific coast of South America, that you would need to fly in a southwesterly direction. Look at the map. Is this true or false? _____ Which way do you in fact need to fly?

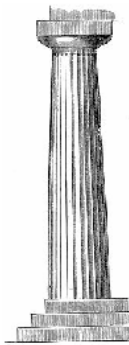
14. And in which direction do you need to fly if you want to go to Rio de Janeiro, Brazil?

15. Now find Scandinavia, in particular, Norway. During World War 2, Norway was occupied by the Nazis, while the Soviet Union, Great Britain, and the United States were allied in fighting Nazi Germany, Japan, and Fascist Italy. It was very important for the combined war effort against Hitler for the United States to send aid to the Russians – no small feat, considering how far apart the two nations were, and the obstacles in-between. One of the major routes for US aid to the Soviets was via merchant marine ships, sailing past Iceland (can you find that?) and around Nazi-controlled Norway, but avoiding the pack ice of the

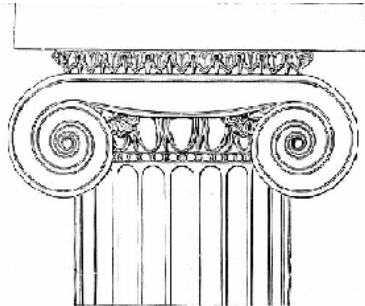
Arctic Ocean, to Russia’s ice-free port on the Karelian Peninsula at Murmansk. You may or may not know that Murmansk is north of the Arctic Circle, as is northern Norway; thus, at this time of year (mid-December), the entire passage was made in the dark. Still, the ships had to avoid both U-Boats (German submarines) and aircraft based in Norway. Many ships and men were lost during the war, both in supplying Russia, Great Britain, and in organizing the successful landings in Morocco (northwest Africa – Operation Torch), Normandy (Operation Overlord – northern France) and others.

E. PENNSYLVANIA AVENUE – BETWEEN THE ARCHIVES METRO STOP AND THE NATIONAL GALLERY OF ART

- At the exit from the Archives Metro station, look up at the buildings around you. To your north is a semi-circular commercial building – divided into two parts by a passageway and then a street – that has a lot of columns on it. Look carefully at those columns. Now look to your south, across Pennsylvania Avenue. You should see the National Archives building. Once again, it has columns. Look at them closely, too. Now look to the southwest across 9th Street, diagonally across the corner from the Navy Memorial, and notice those columns as well. Now look at these examples of the three main, traditional types of columns in the Greek and Roman world. (This type of monumental architecture with columns and beams was borrowed from the Egyptians.) Which building has which?



Doric Order



Ionic Order



Corinthian Order

- When the Greeks built the famous Temple of Athena (known as the Parthenon) in Athens, they wanted to avoid having the columns appear to be concave, which is how they would appear if the sides were straight. Therefore, they made them a bit convex – thicker in the mid-section, so that they would appear straight. They even had them lean in slightly different directions, for the same reason. Can you tell whether the modern DC architects did the same thing? _____

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3. Geometry students: Notice the numerous friezes, or strip patterns, all around you on the buildings. For your chapter project, you will need to find many examples of these strip patterns and to invent some new ones. (Beware – what classical architects refer to as ‘friezes’ are not what our textbook intends! Thus, the triangular area over the front entrance to the Archives is often referred to as a ‘frieze’ in architectural terms. But when we discuss friezes in our class, we are talking about strip patterns.)
 4. Speaking of fascism – the word comes from ‘fascies’, a bundle, and is usually represented by an old Roman image of an ax-blade protruding from a bundle of sticks or rods. Can you find one of these unfortunate symbols on the side of the base of the statue representing “Study the Past” in front of the northern entrance to the National Archives?
 5. The sidewalks of Pennsylvania Avenue are made of square, brown bricks. They form one of the three regular (or ‘pure’) tessellations. What is the numerical code for this tessellation? _____
 6. Notice also that there are at least two different styles of streetlights along Pennsylvania Avenue. One is a fancy type known as ‘acorn’ light fixture, found in many well-to-do and historical neighborhoods. These lights cast their rays pretty much in all directions – including towards the eyes of motorists, and straight into the air. However, there are also some other fixtures that only cast their light downwards. The latter are on curved supports, and are known as ‘full cutoff’ lighting. Many lighting experts believe that full-cutoff fixtures save a lot of money by directing the light where it is needed – on the ground, and not into the air, and they also actually improve visibility at night because they reduce glare. They have the additional benefit that, if they are used widely, they permit people to view the stars, planets and even the Milky Way at night, which is now almost impossible to do anywhere within 20 miles of the White House. Your opinions? _____
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7. Examine the friezes on the metal window guards on the eastern (rounded) end of the Federal Trade Commission building at the corner of Pennsylvania and Constitution Avenues. There are different geometrical patterns there – a strip pattern along the left and right-hand edges, and a pattern of diamonds (rhombuses) in the center. What sort of symmetry does each one have? _____
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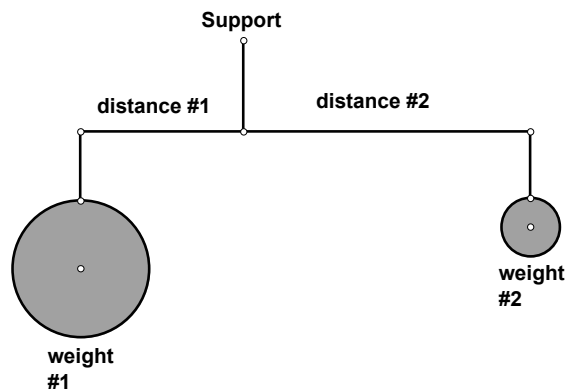
F. NATIONAL GALLERY OF ART – EAST WING

This museum is located on an oddly-shaped piece of land – a trapezoid with two right angles, formed by the intersections of 4th Street, Pennsylvania Avenue, Constitution Avenue, 3rd Street, and Madison Drive. It was quite a puzzle to figure out what sort of building would fit and would somehow be modern and would also match the older, West wing of the National Gallery of Art, built in the late 1930’s and opened in 1941. The famed architect I.M. Pei won the contract to design the building, and he incorporated the angles of the piece of land into the building itself, which opened in 1978.

1. Notice that the entire floor is marble isosceles triangles. Use the protractor that your group was given to measure the angles: _____ and the lengths of the sides: _____ (give the units you are using!)
2. Can you find any right angles anywhere on the floor itself of this building? _____ If so, you certainly had to look hard. Where did you find one?

3. Get a copy of the small guide to the National Gallery, and look at how each floor of the East Wing is laid out. Then look around at the building as a whole. My understanding is that since there were so few right angles in the building (even in the bathrooms!) it made it a very, very expensive building to construct. My own experiences with home improvement tell me why they call 90-degree angles “right” angles – because they really are the right ones to build with – they make everything so easy. When you have angles other than 90 degrees, everything is much more difficult to build, and there is a lot more waste material that ends up being thrown away.

4. If you are in the main part of the East Wing, you will see a ‘mobile’ – or moving, hanging sculpture by Alexander Calder. It weighs several **tons**. Calder originally got a degree in mechanical engineering, and then turned to art. At first, he used to work out the balance points for his mobiles mathematically, but later he ended up doing them ‘by eye’ and by trial and error, using pieces of metal, wood, or whatever looked right to him. Let’s do a little bit of math, as Calder may have done on his early works. All we need is the basic principle of the lever or fulcrum. We will pretend that we have horizontal rods that don’t weigh anything at all, and that the wires and other fasteners don’t weigh anything either. Examine the diagram to the right. The basic principle is :

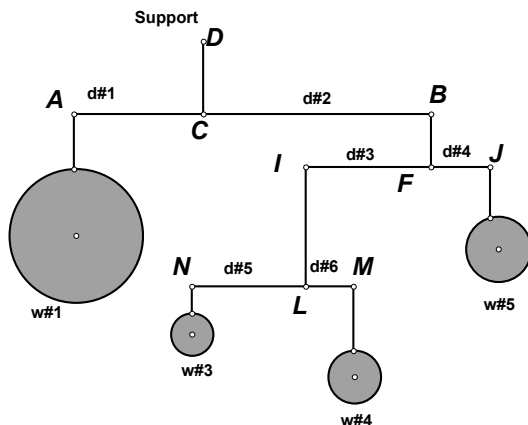


$$(\text{weight \#1}) * (\text{distance \#1}) = (\text{weight \#2}) * (\text{distance \#2})$$

5. Or, to abbreviate, $w_1 \cdot d_1 = w_2 \cdot d_2$. So if distance #1 is 6 inches, and weight #1 is 12 pounds, and distance #2 is 18 inches, then what would the second weight have to be?

_____ (*1)

6. That was probably pretty easy. But as you can see from the mobile above your head, it can get much more complicated. Here is another idealized mobile:



7. You may be relieved to know that the total weight at point I is simply $w\#3 + w\#4$, and the total weight at B is simply $w\#3 + w\#4 + w\#5$. If you know that $w\#3 = 2$ pounds, say, and that $w\#4 = 4$ pounds, it is fairly easy to see that $d\#5$ must be twice as long as $d\#6$. And if $w\#5$ is 8 pounds, then what must be the ratio of $d\#3$ to $d\#4$?

_____ Once you have all

that worked out, then, if you know how much $w_{\#1}$ is, then you can work out the ratio of $d_{\#1}$ to $d_{\#2}$.

8. In practice, however, it's a bit harder. For one thing, those horizontal bars do have weight, obviously, and so do the strings. And even if you get it all worked out perfectly in theory, then minor changes in where you drill holes or tie cords may throw things off. Thus, there is a good bit of trial and error in the construction of any mobile, though simple algebra will help in getting you started.
9. Outside corner wall of East Wing, National Gallery, southwest corner (towards Madison Drive and 4th street): This is the sharpest angle on any wall that I know of. See if you can measure the angle using the protractor and a ruler. Is it one half of the smallest angle on the isosceles triangles on the floor of the main building? _____ Notice the many hands that have rubbed the wall. Why did they do that? Probably for the very same reason you wanted to rub the wall

G. SPECIAL M.C. ESCHER EXHIBIT

1. Before viewing the special exhibit of prints and drawings by M.C. Escher, you must first check all of your bags, coats, paper, books, and writing instruments in the cloakroom that is to the right (south) of the ground-floor entrance of the museum. Your group can probably take along one copy of this study guide, though the curator may perhaps have you leave it outside the door to the study room just before entering. Remember that these prints are extremely valuable! Be very gentle, and don't touch anything that you are not supposed to touch!
2. The study room is on the 3rd floor of the museum, in an area that is normally off-limits to the public. To get there, head past the sculptures on the main floor, past the stairs that lead to the concourse (or basement) level, and past some trees. You will see a glass wall with a door. Enter the doorway, and tell the guard that you are going to the special study room for viewing prints, and that you have an appointment to see Mr. Greg Jecmen to view the prints and drawings by M.C. Escher that were put on display by him for Mr. Brandenburg's students at Alice Deal Junior High School. The guard will probably have you sign in (and may, perhaps, ask for some identification for the adults), and will show you the way up.
3. I am not exactly sure which prints Mr. Jecmen will have on display for us, but I suspect that they will be selected from the following list:
 - One of his prints entitled "Circle Limit III" from 1959. This is somewhat similar to the tessellation in my classroom with the donkeys. It uses hyperbolic geometry on a Poincare disk, and as you go closer and closer to the edge of the picture, the tessellating figures get smaller and smaller, until they reach an infinitely small size and, as a limit (a term you will learn in pre-calculus) they have no size at all. How on earth did he do that? By the way – in hyperbolic geometry, all lines are circles, and they intersect the edge of the Poincare disk at right angles.
 - "Day and Night" from 1938 and is a very, very famous print. A calm land- and water-scape in a Dutch village blends day into night, as black squares turn into white ones and white birds turn into black. How on earth did he do this?
 - "Eight Heads" dates from 1922 is a less well-known print, but, as far as I can tell, it is the first one where Escher worked out a way for different figures to tessellate – or to tile the plane. It is said that his earliest exposure to the idea of tessellation was

when he visited the Alhambra in Granada, Spain, but I don't know off-hand when that visit was.

- “Metamorphosis I” (1937) and its siblings “Metamorphosis II” (1939) and “Metamorphosis III” (1967-1968) are, again, very famous. Notice how each little figure, alone, could tessellate. However, to make it more interesting, Escher subtly changes one figure into another one over a period of several images. Metamorphosis II and III are huge prints, and they required many sheets of paper and many blocks of wood to create. Either one of them would probably fill a wall.
 - I don't think that I have ever seen “Plane-filling Motif with Birds” (1949) or “Plane-filling Motif with Fish and Bird” (1951), so I don't know how to describe it/them. Try to figure out what types of symmetries are shown, and what transformations take you from one figure to the next.
 - Similar remarks for “Regular Division of the Plane” versions II, III, IV, V, and VI (1957).
 - I think you will also get to see “Smaller and Smaller” (1956), but I haven't seen this one myself. Again, try to figure out the symmetries and transformations.
 - You might not get to see “Snakes” (1969), but it is really beautiful.
 - You might get to see “Square Limit” (1964), and it is possible that it might be similar to Circle Limit. Can you explain what is going on in this print?
4. Take mental notes on each of the prints, and examine them carefully. However, since we only have 2 hours total to view them (10AM to 12 noon), and we have 10 groups, you will have to be quick. 120 minutes divided by 10 groups is only 12 minutes per group. Use your time wisely!
 5. Mr. Jecmen was one of the people who arranged the 100th anniversary exhibition of the works of Escher here at the National Gallery about 5 or 6 years ago. He can probably answer any artistic questions you might have about Escher's work, and may have some corrections that he would like to make to some of the information in this guide.
 6. Please be sure to thank him for arranging this behind-the-scenes tour!
 7. When you finish here, you may get the coats, bags, and other things that you checked in the cloakroom, and then proceed to the next part of the tour.

H. TROMPE-L'OEIL EXHIBIT EAST WING, NATIONAL GALLERY OF ART MEZZANINE LEVEL

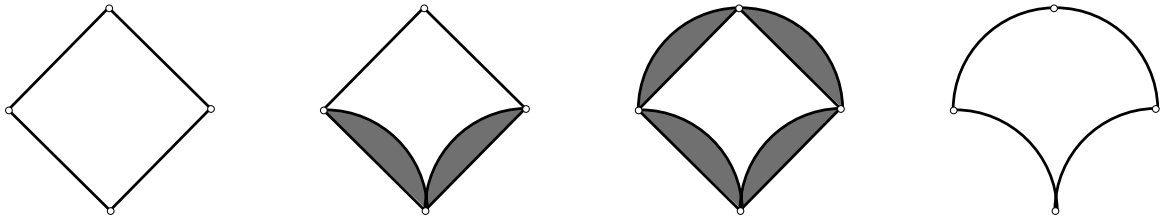
You may perhaps have time to view a small part of this exhibit, which is on using art to fool the eye of the beholder into thinking that something painted really is real. In French, “tromper” means “to fool” and “oeil” means “eye” – thus, “Trompe l'oeil” means “Fool the eye.” Some of the exhibits really do a good job of doing that! They require a good use of perspective – first studied mathematically during the Renaissance, as far as we know – but also a few other things. Even the ancient Greek and Roman artists tried their hand at this process, as you will see if you look at the mosaic on the floor at the very start of the exhibit. Just don't annoy the guard who is standing there by trying to touch anything, no matter how real it looks. Wait Oh, I see. I was fooled! (Were you?) _____

If your group does take a brief detour through this exhibit, then list the items that really did fool you:

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

I. STREET-LEVEL PLAZA BETWEEN WEST AND EAST WINGS, NATIONAL GALLERY OF ART (4TH STREET)

1. Notice the pink granite cobblestone pattern on the plaza. This is a tessellation. You can think of it as having originally been a square (or rhombus), but then an arc segment got removed from two adjacent sides of the square and rotated towards the unaffected sides – or else translated (slid) across to the opposite sides, as you see here:

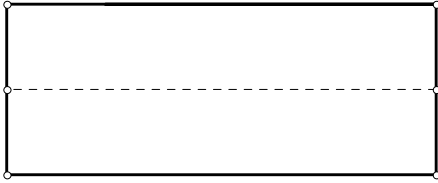


2. If you observe carefully, you will notice that the cobblestone pattern changes direction in the middle of the street. But don't stand there long! Watch out for cars!
3. We perhaps could estimate the weight of one of these cobblestones. They are about _____ cm wide and _____ cm across and _____ cm deep (into the ground – you will simply have to make a guess on that dimension). Their volume is simply width X length X depth, so you have a volume of roughly _____ cubic centimeters. The specific gravity of granite is roughly 3, which means that if you multiply the volume by 3, you will get the weight in grams. Result: _____. To convert that into kilograms, simply divide by 1000. Result: _____ (feel free to round everything off to make the math easier) And if you want to convert that into pounds, multiply the last answer by 2.2. Weight, in pounds: _____
4. In Paris, as in many other European capitals, streets were normally paved with cobblestones, because they are cheap and easy to find and last a long, long time. Unfortunately for whoever happens to be in power, when an angry mob starts looking for something to throw at police and other authorities, cobblestones fit the bill – and the hand – rather well, though you can't throw them like a baseball. Plus (or so I was told by people who have done it), it is really hard to get the first cobblestone out (after that it's pretty easy, they say, since there is no mortar holding them together). So, since the worker-student uprising of 1968 in France, the authorities there have paved over just about all of the avenues in Paris with asphalt.
5. On the other side of 4th street, towards the West Wing, are a number of strange objects that were also designed by Pei. There is a circle that is formed by a whole lot of pink granite bollards – cylinders with hemispherical tops. How many of the bollards are there?

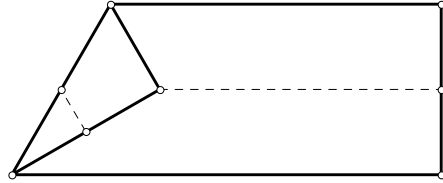
- (I counted the end of one of the low walls as one of the bollards.) _____ Does that number have anything special to do with circles, that you know of? _____ How many degrees are there between each bollard? _____
6. By the way, during the French Revolution, there was a strong move to make absolutely everything decimal, which these bollards are not. They decided to use 100 ‘grads’ in a right angle – 400 ‘grads’ in a circle—and they even had 10-day weeks (highly unpopular with the peasantry, because they had fewer days to rest). They even had a 10-hour day with 100 minutes per hour and 100 seconds per minute. That works out to how many seconds per day? _____ We, on the other hand, have 24 hours per day, 60 minutes in an hour, and 60 seconds in a minute, all of which we probably inherited from the Babylonians. So how many seconds do we have in a day? _____ Which system had the longer second? _____ Which system would you prefer? _____ Why? _____
7. The very layout of the streets of Washington, DC – including the broad streets meeting at right angles, the avenues that connect focal points – circles and other important points, going off at other angles, and the National Mall itself are of course the creation of a Frenchman, Pierre L’Enfant, who fought bravely in the Revolutionary War for the cause of liberty and reason. His ideas, like the ideas of Thomas Jefferson, Benjamin Franklin, George Washington, and many others came out of the ideas of the Enlightenment of the 18th century. If you visit any of the major palaces and royal parks in France or elsewhere on the European continent (Fontainebleau and Versailles come to mind) you will see the same broad malls and avenues, with important buildings like the U.S. Capitol situated on important hills. The French enlightenment also gave us the metric system, which was begun under the monarchy brought to fruition during the French Revolution and the Napoleonic era, and almost reversed during the Restoration.
8. What’s more, the study of geometry and of logic even led to the ideas behind our Declaration of Independence. If you re-read the Declaration carefully, you will notice that it is written in the form of a proof. It starts out with a general statement of what the signers are trying to prove. Then follow several axioms, or postulates, that the writers assumed to be true – one of those being, that all men are created equal. After a few more postulates, the writers then went on to prove that the King of England had violated a large number of natural laws, and that the United States of America was right to declare its independence.
9. Inside the circle of bollards are water fountains and a number of tetrahedrons, or tetrahedral – 3-dimensional shapes whose faces are all triangles – triangular pyramids, if you like. The tetrahedrons act as skylights to the cafeteria and walkway below, where you will eat lunch (or have already eaten lunch). These are not regular tetrahedrons, because the faces are not equilateral triangles. Regular tetrahedral are one of a very small number of regular polyhedrons (five, to be exact)—ones whose faces are all congruent regular polygons (those with all congruent sides and all congruent angles) and at whose vertices you have the exact same arrangement of polygons. The very last proposition of Euclid, following his very last “proposition” (Book 13, proposition 18) goes as follows: “I say next that no other figure, besides the said five figures, can be constructed which is contained by equilateral and equiangular figures congruent to each other.”
10. Those figures are: tetrahedron (four faces, all triangles), the cube (6 faces, all squares), the the octahedron (8 faces, all triangles), the dodecahedron (12 faces, all pentagons), and the icosahedron (20 faces, all triangles).

11. It is easy to make a regular tetrahedron with a dollar bill, if anybody has one. {Relax – you will get it back.} Fold it exactly in half the long way, and crease it. Then straighten it back out. Make a fold so that one corner of the bill comes right down to the center line you just made; the other end of the fold is the other, nearest corner of the bill. After the first two folds, your bill should look like this:

Step 1



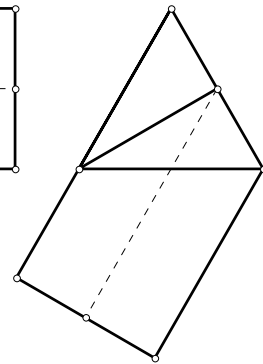
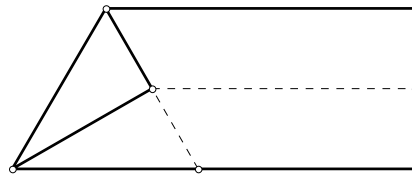
Step 2



12. Next you continue folding the bill into

equilateral triangles, using the edges you have just created:

13. Eventually it will all fold into a very nice, compact equilateral triangle, whose edges you should crease. Now unfold the bill and carefully rearrange it – you should be able to make it



stand up like a triangular-based pyramid, or regular tetrahedron. Were you successful? _____ And if you fold it so that the ‘back’ of the dollar bill is on the outside, you are likely to have the All-Seeing Eye on top of the pyramid looking right out at you!

14. Remember that the Concourse Café and the passageway from the East to the West Wing of the National Gallery of Art is right underneath this plaza. The builders (and the architect) did an excellent job in designing the plaza and the concourse so that they don’t leak, unlike many similar structures!

J. WEST WING OF THE NATIONAL GALLERY

We don’t have a lot of time to spare in the West Wing, but you should by all means look at a few things:

1. The stairs and landings leading from the ground level up to the main floor are made of marble that contains – in places – easily visible fossils of shellfish of various types.
2. Gallery 66 contains a plaster model of the heroic statue by Augustus St-Gaudens honoring Robert Gould Shaw and the heroic Massachusetts 54th Regiment that was also immortalized in the movie ‘Glory.’ The bronze version of this statue has been on display on Boston Commons for about 105 years, but this model has been displayed in various locations at various times. It was quite a geometrical and mathematical puzzle to figure out how to make it in the first place. Have you seen that movie? _____ Do you

remember why this monument was made, and why that regiment was so important to American history? _____

3. At the exact center of the National Gallery, on the main floor, is a very large rotunda and dome, surrounded by a circle of columns. Which order do the green marble columns belong to? (See section D of this guide for a quick refresher.)
_____ How tall do you think the top of this dome is?

4. There are three very well-known, large domes nearby – the U.S. Capitol dome, the Library of Congress, and the West Wing of the National Gallery. You can see a huge aerial photograph that includes all three domes on one of the landings of the escalators that lead from the concourse level (underground) to the ground level of the gallery. These are all more-or-less modeled on the Pantheon in Rome, which was the first really big dome to be built. All domes are ‘surfaces of revolution’ – and, if built properly, allow the public to gather inside in a large space with no columns getting in the way. The Pantheon in Rome was built with concrete (!!!!) which the Romans had invented. After the fall of the Roman Empire, the secret of making concrete was lost for many, many years. The forces of gravity on a dome tend to make it want to fall apart towards the outside, however. Thus, the thickness of the walls of the Pantheon dome are measured in yards (meters), not feet. However, the U.S. Capitol dome was built with a much lighter and stronger material – cast iron. Chances are that the dome here in the National Gallery also has a steel or iron skeleton.
5. At some point, you might want to visit the public part of the Library of Congress. There are more geometric designs, tessellations, friezes, and mosaics than you could count in a day in that building, but we don’t have time today.
6. Notice the tessellating pattern that you see on just about every air vent in the West Wing of the National Gallery. Please make a sketch in the space below of one of the repeating parts of the tessellation.

K. WALK FROM THE WEST WING OF THE NATIONAL GALLERY TO THE SMITHSONIAN’S MUSEUM OF NATURAL HISTORY

1. If you look to your south, towards the other side of the Mall, you will notice the Smithsonian’s National Air and Space Museum (NASM). Don’t go there now, but along the north side (facing you) of the NASM, and continuing past the Hirschhorn museum and the Smithsonian’s Castle, is an accurate scale model of our solar system. This is not like the one you might have made, drawn or seen in your geography or earth science class, where the Sun is a ball about 2 inches across, and Mercury is a marble about 3

inches away, Venus and Earth are some more marbles about another inch away, and the whole thing fits on a table top. No, this model is really accurate. So, for example, the Sun is a yellow ball about the size of a tennis ball or an orange, Mercury is a tiny dot about 20 feet away, Venus is another tiny dot about 40 feet from the Sun, and Earth is another tiny dot about 20 more feet further still. The Moon is an almost invisible dot about 2 inches from the Earth. Jupiter and Saturn flank the north entrance to the museum – nearly half a football field from the Sun – and they are the size of marbles. To get to Pluto (another tiny dot), you have to walk towards the Washington Monument for a couple of blocks (I've always run out of energy before getting there, myself). Where do you think the nearest star might be? Before or after the Washington Monument? _____ Inside DC? _____ In Virginia? _____ West Virginia? _____ The answer will almost undoubtedly surprise you. (*²)

2. Unfortunately, most of the models of the planets have currently been removed for repair. (That's another reason not to go and look at them right now.) However, it so happens that tomorrow, December 17, 2002, will mark Saturn's closest approach to the Earth for the past 30 years. This also means that Saturn rises just about sunset and sets about dawn, and is as high in the sky as possible around midnight. What's more, its rings are tilted as far as they can get. So if you (or a friend) have a telescope, and the weather clears up, this vacation would be a good time to take a look at this planet. Have you ever seen it before with your own eyes through a telescope? _____
3. You may know that Galileo Galilei was the first person to use a telescope to look at any of the planets or the Milky Way. He could not figure out what the rings on Saturn were – his telescope was horrible by our standards, and he thought that they looked like ears or handles on a jug. Then, the weather got bad for over a month, during which both Saturn and the Earth changed locations, and he was unable to look at Saturn for several months. When Saturn became visible again, the rings of Saturn had changed their tilt as far as the Earth was concerned, and when Galileo went to show skeptical authorities the 'ears' on Saturn, the rings were invisible, since they were tipped edge-on towards the Earth.
4. He also was the first person to see the moons of Jupiter. He was amazed to find that the moons would visibly change their locations even in the course of a couple of hours! This was really the first proof that not everything in the universe revolved around the Earth, contradicting the theories of Ptolemy that were upheld by the church authorities, and getting Galileo into a lot of trouble with the Roman Catholic Church. (He was investigated under the Inquisition, convicted of heresy, and sentenced to house arrest for the rest of his life.) One of the major ironies of all of this was that Ptolemy, whose astronomy Galileo was criticizing, was a pagan and perhaps had never heard of the Christians, while Galileo always thought of himself as a defender both of Christianity and of reason. Have you ever seen Jupiter or its moons in a telescope? _____ Even very steady binoculars will show them.
5. Galileo was also the first person to see the phases of Venus. It changes its shape, and its size, to us viewers here on earth, much like the Moon does, but much more dramatically. He felt, correctly, that this was another proof that the Sun was the center of the solar system. Right now, before the Sun rises, Venus is extremely bright and high in the eastern sky. Have you noticed it? _____
6. You will see a first edition of Galileo's great book, "Siderius Nuncius" or "Starry Messenger" just outside the Dibner library at the Smithsonian's Museum of American History. The book is open to a page with a drawing by Galileo that shows that the Milky Way is not just a cloud, as everyone used to think, but is actually made up thousands of stars.

7. You will pass by the National Sculpture Garden, which also contains a popular ice-skating rink. We don't have time for you to spend much time looking at the sculptures, but many of them (towards the south, or Mall, side) are quite geometrical. There is one in the shape of a square pyramid, made out of concrete blocks, and another which uses perspective to give the illusion of a 3-dimensional house. Another sculpture is composed of stainless-steel cubes that are perfectly balanced and put on ball-bearings, so that even the slightest gust of wind will make the cubes rotate. Do you see any sculpture(s) that you particularly like? _____ If so, then which one(s) _____

8. Notice that there is a frieze, or strip pattern, along the bottom edge of the fence surrounding the sculpture garden. Try to sketch it, and tell what type of symmetry it has:

9. In addition, the gates of the sculpture garden also have a frieze/strip pattern, but this one goes up and down, rather than left to right. Sketch this frieze, and describe its symmetry as well.

L. SMITHSONIAN'S NATIONAL MUSEUM OF NATURAL HISTORY

We don't have a lot of time to spend in here, but we should look at just a few things: the geometry in and around the main rotunda, and part of the national collection of gems and minerals.

1. The main rotunda of course holds the enormous elephant, but we are here to look at other things. First of all, we have yet again another dome. What order (if any) are the columns that hold it up? (See part D if you have forgotten the orders. Walk around the alcoves surrounding the rotunda, and look down at the floor and up at the ceiling. Are these tessellations or friezes, or both? _____
_____ Sketch one or more of them, and describe its symmetries:

2. Look up at the roof of the dome, and notice the four arches. Each arch has an arrangement of octagons and squares – one of the 8 semi-regular tessellations. Can you figure out what its code is? _____ (*³)
3. Proceed by stairs to the second floor. Stay in the main rotunda area. Notice that the banisters along the stairs again form a frieze, but we are running out of time, so you won't sketch them. Go to the entrance to the gems and minerals section labeled "Hope Diamond." After you have admired it for a while, then turn right and look at some of the amazing crystals and gems in the next room. Their beauty is amazing, though most of them have no use other than the fact that people find them pretty. (Diamonds are an exception – their extreme hardness means that little diamond chips are extremely useful for grinding, drilling, and polishing other things. Rubies used to be useful in the production of lasers. Absolutely pure – and hence artificial—silicon and germanium crystals are the basis of the entire electronics and computer industry of today. But nobody would want to wear one!)
4. The great flawless quartz crystal ball is said to be the largest in the world. It has a diameter of about 32.7 cm, and weighs about 48.5 kilograms or about 48,500 grams. Using this information, we can compute its specific gravity – i.e., how heavy it is compared to water (which, by definition, has a specific gravity of 1). The formula for the volume of a sphere is $\frac{4}{3}\pi r^3$. The radius of half of the diameter, which means it's about 16.35 cm. You probably don't have your calculator with you, and this calculation would not be a lot of fun to do in your head or on paper, so I've done it for you. I get about 18,300 cubic centimeters for the volume. Now all you have to do is to divide the weight in grams by the volume (remember – round things off to make the arithmetic easier) and you get a specific gravity of _____. In other words, this sphere is about _____ as heavy as a ball of water that has the same dimensions. (*⁴) So it will definitely sink to the bottom of the ocean if it got the chance.
5. A bit further on, you see a little display on common table salt, or NaCl. You may not realize it, but salt is a crystal, too – only it's so abundant, and so soluble in water, that it's not very valuable these days. (But in the old days, parts of the world that didn't have salt were in very great trouble. It was one of the few things that helped heal wounds, and it also was absolutely essential in preserving meat and other foods. Also, without salt, you die a painful death. And without sea salt [with its iodine impurities] you got goiter, a horrible deficiency disease of the thyroid gland. Kings were very fond of having monopolies on the manufacture of salt, because it was a guaranteed source of revenue. In fact, the word "salary" comes from the Latin word for salt. There are sayings such as 'not worth his salt.' Maybe the adults in your group might know some more such sayings.)
6. The big deal about crystals is that they are made up of an almost perfectly uniform arrangement of atoms (or ions) in a solid, 3-dimensional lattice. Some of the atoms are larger than others, and some of the bonds between the atoms are stronger than others, which means that the structures of the lattices are different for different minerals or crystals. If you have ever seen somebody stacking a whole lot of identical oranges on top of each other to form a nice display pyramid at your grocery store, then you have an idea of what one such arrangement might look like. But what if you were trying to stack equal

numbers of ping-pong balls and tennis balls? It gets a lot more complicated, and you might need glue or toothpicks to get them to hold together. And what if it's two ping-pong balls for every tennis ball, and one orange for every 3 ping-pong balls, and they have to always be in such and such an arrangement with each other?

7. Well, I can't really visualize it exactly either, not without trying it myself. But it so happens that there is a relatively small number of different possible arrangements of crystals, depending on the number of axes in the 3 dimensions, the relative lengths of those axes, and the angles that the axes form with each other. Crystallography is a fairly new science. One of the ways that scientists discover the structure of crystals and molecules (such as DNA) is to send X-rays through the crystal lattice. The X-rays interact with the atoms in the lattice in particular ways which depend on the angle that they hit the atoms, and then scientists examine the spots that are formed on the photographic plate on the other side of the crystal. Naturally, a lot of mathematics is involved in interpreting these X-ray diffraction patterns.
8. Here are the basic crystal systems:
 - Cubic: Three crystal axes of equal length intersect at right angles to each other. e.g. diamond, spinel, garnets.
 - Tetragonal: Three axes intersect at right angles to each other. The vertical axis is of unequal length while the two horizontal axes are of equal length. e.g. zircon, rutile.
 - Hexagonal: Four crystal axes. Three are of equal length and intersect at 60° to form a horizontal plane which the fourth intersects at right angles. The vertical fourth is of unequal length and forms an axis of 6-fold symmetry. e.g. Beryl, apatite.
 - Trigonal: Four crystal axes. Three of equal length intersecting to form a horizontal plane which is intersected at right angles by the fourth axis. The vertical fourth is of unequal length and forms an axis of 3-fold symmetry. e.g. quartz, corundum, tourmaline, diopside, hematite.
 - Orthorhombic (Rhombic): Three crystal axes of unequal length intersect each other at right angles. e.g. topaz, peridot, chrysoberyl, iolite, sinhalite, andalusite.
 - Monoclinic: Three axes. Two of unequal length intersect each other obliquely to form a plane which is intersected by the vertical third (of unequal length) at right angles. e.g. jadeite, nephrite, diopside, orthoclase feldspar, serpentine, sphene, malachite, spodumene.
 - Triclinic: Three axes of unequal length intersect each other at oblique angles. e.g. turquoise, labradorite.

Of course, there are sub-divisions and gradations of these. Check off, or circle, the ones on this list that you find in the display. Again, don't spend too long here – we need to get to the Smithsonian's Museum of American History (MAH).

L. SMITHSONIAN'S MUSEUM OF AMERICAN HISTORY

1. If you enter by the mall (south) entrance, you will notice a shiny metal sculpture just outside the doors. It slowly rotates, and is much like a Moebius strip, in that it only has one side, even though it looks like it has a triangular cross-section, and hence, 3 sides.

What I mean is that if you climbed up onto the pillar with a crayon and started drawing at any location on the strip, and kept on going long enough, you would eventually mark up the entire length of the sculpture – all sides – and come back to the beginning point, if the guards didn't drag you away first. Have you ever messed around with a paper Moebius strip before? _____

2. If not, too bad. They are fun. All you need is a strip of paper about 12 inches long and 2 inches wide, like the ones you used to make in elementary school to make decorations and paper mats. You make a single loop, but before taping the ends together, you make a half turn with either one of the ends. Now tape it together. You probably didn't believe me about the sculpture, and you certainly can't actually try it out, but it is perfectly OK to take a crayon, pen, or marker and start marking one side of the paper Moebius strip you made. You will find that you end up marking the entire strip – what appears to be both 'sides' (but there is only one 'side!') – and coming back to the start.
3. The fun has only just begun. Now take a pair of scissors and cut the strip in half lengthwise, going all around the entire thing. You probably expect it to fall apart into 2 strips. Surprise! And then try cutting what you have left in half again. Surprise again!
4. Go to the ground floor and look or ask for the Dibner Library, located a bit west of the exact center of the building, near the entrance to "Information Age." Outside the Dibner Library are numerous items of interest, which you should look at if your group is waiting to enter the library itself. They are mentioned in items 5, 6, and 7.
5. A large display case is entitled "Slates, Slide Rules, and Software: Teaching Math in America." It contains numerous visual and other aids that have been used to teach math in this country over the past 150 years or so. What items in this display do the **students** in your group recognize?

____ What items in this display do the **adults** in your group recognize, that the students do not?

—

____ Is there anybody in your group who knows how to multiply 2 times 3 on a slide rule? _____ (Hint – it's all based on adding logarithms.)

6. If you still have time, quickly go through the Information Age exhibit – it takes you from the first telegraphs all the way up through the first electric data-handling machines through the code-breakers of World War II, up to many aspects of robotics and modern computers.
7. Inside the hallway that leads to the Dibner Library is a collection of very valuable first editions of famous books. Can you find the following?
 - A book Albrecht Dürer (1511), the Renaissance-era German artist and engraver who was extremely interested in the geometric and mathematical aspects of perspective, shadows, and so on. This book was absolutely ground-breaking and was studied by artists for hundreds of years.
 - The 1511 translation of Ptolemy's Almagest, mentioned in section C of this guide. This book gives a very convincing argument for why the earth must be spherical, and for why it cannot possibly revolve around the sun. (What evidence can **you** give, if you were challenged, and only based on your own personal observations, that the earth is not flat and that the earth rotates on its

own axis and orbits around the sun?) This volume is open to one of the first maps which shows part of the New World that Columbus ran into.

- A book by Carl Linnaeus on the classification systems for plants and animals. You may know that he was the person who invented the binomial (genus-species) method of naming plants and animals, and devised the basic structure of kingdoms, orders, families, and so on that we use today.
- A book by Robert Hooke, a pioneer in the study of cells, on what he saw through a microscope.
- A book by Hevelius on astronomy.
- A book by Robert H. Goddard (for which NASA-Goddard is named) on rockets.
- Kepler’s 1596 book describing the way that he thought the Solar System was formed, based on the relative sizes of regular polyhedra (mentioned in section H of this study guide). Some of his ideas were right, and others were wrong.
- A 1496 translation of Ptolemy’s Almagest, mentioned earlier.
- A book by Isaac Newton on optics. Newton invented both calculus and the reflecting telescope, after deciding (incorrectly) that it was impossible to correct the lenses of refracting telescopes for the fact that they tended to turn all bright objects into rainbows.
- Siderius Nuncius Magnus (“The Great Starry Messenger”) by Galileo – the book where he first proclaimed his observations of Jupiter, its moons, Saturn, the stars of the Milky way, and much more. See section J for more details.
- A first edition of Charles Darwin’s The Origin of Species – the absolutely ground-shattering book on which all of the study of life on earth (biology, botany, zoology, etc.) is based today.

9. If you have additional time, then look into the Information Age exhibit. Some of the items will be ones you have heard of, but others might be completely new to you. Did you know that IBM, or International Business Machines, got started right here in Washington, DC, as the Tabulating Business Machine company? It was organized by inventor Herman Hollerith to sell machines to the Census Bureau so that it could count the results of one of the every-ten-year national census. List the items that you actually recognize or have heard of in this huge exhibit:

_____ Don’t miss the original many-hundred-page proof of the famous 4-color theorem, towards the end of the exhibit. (*³) How many of the various computers on display do the parents in your group recognize, that the students don’t? _____ Which ones?

-
10. If there is time, then investigate the exhibit on Invention near the northern (Constitution Avenue side) entrance to the museum. In particular, investigate the kaleidoscopes – or, should I say, the hinged mirrors. You will find that certain angles permit you to see an entire circle of reflected objects, but other angles don't. Which angles work perfectly?
-

- Can any of the geometry students explain why that is so? _____
11. On the upper level of this museum is an exhibit called “On Time.” It contains many early clocks, as well as an enlarged facsimile of one of Benjamin Banneker’s almanacs. You may know that he was a free African-American farmer living in the area of what is now known as Ellicott City, MD, and taught himself a great deal of mathematics and astronomy, using books, telescopes, and other instruments lent to him or given to him by his neighbors, the Ellicotts, who were Quakers – and hence, opposed to slavery. The clock that he made out of wood made him a local celebrity. Later he did all of the lengthy astronomical computations necessary to write an almanac, which is a listing of the locations of the planets during the entire year to come, as well as the correct rising and setting times of the Moon and Sun, along with other information on tides, holidays, and so on. (The silly stuff about the weather forecasts for the entire year was not written by him – that was all added by the publisher.) In addition, Banneker played an important part in the astronomical observations necessary to lay out the boundaries of the Federal City in 1790 – when he was 60 years old! He also wrote a famous, eloquent letter to Thomas Jefferson criticizing him for his racist views about African-Americans. This letter was reprinted in several editions of Banneker’s almanac.
12. Another interesting clock in the “On Time” exhibit shows graphically what portions of the earth are currently illuminated by the Sun, and which portions are currently experiencing the darkness of night-time. Today we are very close to the winter solstice, which means that most of the area north of the Arctic Circle is in perpetual night, and all of Antarctica is experiencing perpetual day. Do you remember those convoys of Allied ships trying to go around Norway to get to Murmansk during World War 2? They never saw the Sun at all during much of the trip. Notice that the boundary between day and night on this map is a curve that rises and falls smoothly – it is called a sine curve, and you will study some of its behavior in Algebra 2 and Pre-Calculus.

M. The Dibner Library – Special Exhibit of Rare, First-Edition Books On Math and Science

Ronald Brashear, who is the Head of Special Collections and Curator of Science & Technology Rare Books of the Smithsonian Institution Libraries, has been kind enough to put together for us a collection of very rare books on mathematics and science. You will undoubtedly have to sign in, and adults may have to show some ID. You will also need to check your bags (and perhaps your coats as well) at the cloakroom, which should be near one of the entrances to the museum. It is a good idea to visit the bathroom right before entering the library, so that you

can wash your hands thoroughly. Dirty hands and old, rare books do not mix well. Here is the listing of books:

1. Euclid. *Elements* [Arabic]. Rome, 1594. It was the Arab civilization that kept alive the knowledge gained by the Greeks and Romans during the time that Western Europe fell into the Dark Ages from about 600 AD to 1300 AD. This book was thought to contain just about everything that was to know about mathematics for nearly 1400 years, and was considered to be a perfect model of logical deduction and proof. I have mentioned before how the Declaration of Independence was written as a logical proof. Abraham Lincoln used to spend a great deal of time studying Euclid, so that he could then demonstrate the theorems himself. If you carefully examine the Gettysburg Address, you will see elements that will remind you of the logical thought processes of Euclid.
2. Euclid. *Elements* [Latin]. Venice, 25 May 1482. This version was printed only about 20 years after the invention of the printing press by Johannes Gutenberg! The first book ever printed was the Bible. Euclid's elements came very soon afterwards!
3. Niccoló Tartaglia. *Quesiti et inventioni diverse*. Venice, 1546. Tartaglia and other mathematicians of his day used to have mathematical contests – one would pose a problem to which he had figured out the answer, daring other mathematicians to solve it.
4. Maria Gaetana Agnesi (1718-1799). *Analytical institutions*. London, 1801. Maria Agnesi was possibly the most important female mathematician of the 18th century, though she is now mostly known for a curve known as the 'Witch of Agnesi'.
5. Sophie Germain (1776-1831). Letter [manuscript]. Between 1796-1831. Ms. Germain was one of the most important female mathematicians of the early 1800's.
6. Carl Friedrich Gauss (1777-1855). *Demonstratio nova theorematis omnem functionem algebraicam rationalem*. Helmstadt, 1799. ('A New Proof on the Theorem of all algebraic rational functions.') Gauss is known for his pioneering work with astronomy, mathematics and magnetism.
7. Leonhard Euler (1707-1783). *Lettres à une princesse d'Allemagne sur divers sujets de physique et de philosophie*. St. Petersburg, 1768-1772. ('Letters to a German princess on various topics on physics and philosophy.') Euler is probably the most productive mathematician who has ever lived – his collected works would fill between 60 and 80 normal-sized textbooks!
8. Leonhard Euler (1707-1783). *Letters of Euler to a German princess, on different subjects in physics and philosophy*. London, 1795. The same, but in English...
9. Benjamin Banneker (1731-1806). *Pennsylvania, Delaware, Maryland and Virginia almanack and ephemeris for the year of our Lord 1792*. Baltimore, 1791. I have examined one of Banneker's almanacs in the Rare Book Room of the Library of Congress. They are very small, and very delicate. I checked out his computations of the Sun, Moon, and planets, and found them to be correct, although his notation for where these objects were actually located in the sky is not what we would use today. Today, if we say that you can find the Moon in the constellation Sagittarius or Ophiuchus, that means that it is actually in that constellation in the sky. All of the constellations of the

Zodiac are of different sizes, so the Sun, for example, spends less time in Scorpius than it does in either Ophiuchus or Virgo. (And I bet you didn't know that Ophiuchus was one of the 13 (!) constellations in the Zodiac!) However, in Banneker's day the system was different. The 'First Point of Aries' was defined as not necessarily as being actually in the constellation Aries, where it used to be in Ptolemy's day, but where the plane of the Earth's equator crosses the plane of the ecliptic – the imaginary plane that almost all of the planets orbit in. And they would count every 30 degrees from that point in the sky as being exactly one 'sign' of the Zodiac. It took me quite a while to figure out what he was doing, but it was strictly in accordance with the principles laid out by all of the astronomical authorities of the day. Of course, when I checked Banneker's computations, I didn't have to do any actual computations myself – all I had to do was to start up one of the many inexpensive astronomy software programs on my computer, set the program for a specific day in, say, June of 1792, and see whether the locations of the planets on my computer screen agreed with the locations predicted by Banneker 210 years ago. When Banneker predicted eclipses of the sun and moon, he had to go through an incredibly laborious process that I cannot begin to describe. The more I learn about what he actually accomplished – starting at age 60 and continuing until his 70's, and all of it almost completely self-taught – the more impressed I am.

10. Diophantus of Alexandria. *Arithmeticonum*. Toulouse, 1670. Diophantus lived and wrote 13 books on mathematics around 200 AD, but only 6 of the books survived. Pierre de Fermat had a copy, printed in 1621. In the margin of one page, where Diophantus discusses the fact that there are lots of Pythagorean triples (that is, whole numbers that are solutions to $a^2 + b^2 = c^2$), Fermat scribbled an enigmatic note: 'It is impossible for a cube to be written as a sum of two cubes or a fourth power to be written as the sum of two fourth powers or, in general, for any number which is a power greater than the second to be written as a sum of two like powers.' Or, in modern notation, "There are no whole-number solutions to $a^n + b^n = c^n$ if n is greater than 2." Did he have a proof of this theorem? We'll never really know, because he wrote another comment: "I have a truly marvelous proof of this theorem, but it will not fit in this margin." And he never wrote anything further on the subject. This proposition resisted all attempts at proof on the part of all of the mathematicians of the world for over 350 years, until, a few years ago, a mathematician by the name of Andrew Wiles, using all sorts of exotic results in all sorts of strange and unusual areas of mathematics, and working entirely alone, worked out an utterly astounding proof that made the front pages of many newspapers. Or, at least he thought he had done so until someone found a few errors. To his credit, Wiles then went back to work and fixed the errors. His proof is quite long, and very few mathematicians have the expertise to understand it (and I am certainly not even really a mathematician, so don't ask me to explain it!). But Fermat's Last Theorem, as it is called, has now been proved. The volume in front of you is the first one where Fermat's marginal notes on Diophantus' original work was reprinted. (Any of you who have actually studied religion in detail will recognize that many essential religious works are printed with commentaries by various scholars – who, oftentimes, had actually scribbled those commentaries to themselves in the margins of their copies.)

You can probably ask Mr. Brashear questions about these volumes, and the rest of the Library. Be sure to thank him for arranging this exposition for us! However, since there are

10 groups who will be examining these books, and we only have 2 hours to do it in (11:30 to 1:30), you can't take very long in this room – only about 12 minutes!

Notes and acknowledgements:

Many, but not all, of the general and specific ideas in this study guide come from the booklet and tour entitled “Math on the Mall,” which is the work of Florence Fasanelli, V. Frederick Rickey, and Richard Thorington, which is about twice as long as this guide. I am most grateful for their contributions. Specific quotes in this guide were taken from Simon Singh’s book *Fermat’s Enigma*, and I took some information from my copy of Euclid, which is the translation with commentary by Thomas Heath. I also copied some information from a website on crystal structures. Everything else is my own writing, but I was inspired by many, many sources, including the free guides that are distributed by the National Gallery of Art and the various Smithsonian museums. I would again like to thank Ronald Brashear and Greg Jecmen for their invaluable help in setting up the special exhibitions of this tour. I would like to mention, again, the inspiration of Dr. Fasanelli, with whom I took a summer course on the history of mathematics this year.

It seems to me that this tour is not a complete waste of time as far as your other classes are concerned, because we have discussed a lot of science (astronomy, physics and biology in particular), history and geography, and you have had an opportunity to read (or look at) some original works in Latin, French, and Arabic. In addition, you have had a good bit of physical exercise – you have walked close to 2 miles, I would estimate!

I would also like to thank, profusely, the parents who volunteered to come along on this trip as chaperones. Without their participation, this trip would not have been possible. I hope they learned something, too – about the topics of the trip and even about how their own children interact with others at school activities.

I will try to have enough copies of this study guide published so that every participant can have one to keep, but it might take a little while for that to happen. --- Guy Brandenburg

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Footnotes and answers:

¹ Four pounds.

² Somewhere out in California! And, so far, in this scale model, humans have only traveled to the Moon and back a few times – two or three inches! Do you think there is any way to get to the nearest star, ever?

³ It would be 4.8.8 or 8.8.4.

⁴ Somewhere between 2 and 3 is reasonable. I get about 2.65 on my calculator.

⁵ This theorem says that any map on a flat surface of territorial boundaries only needs at most 4 colors in order to make it so that no two adjoining territories have the same color. It was known for years that it was probably true, but it took a computer to write the first actual proof of the theorem. In fact, this was the very first proof that could (so far) ONLY be done with a computer. That has changed, for many mathematicians, what it actually means to write a proof.